**Coupled Harmonic Oscillators**

This is a reprisal of the content of corresponding files in the Classical Mechanics folder.

**General formula for Potential Energy given pair-wise interaction**

Let’s say our ionic potential energy comes from the sum of some pair-wise potential, like the Coulomb potential for instance. Then we can say in general,



And we’d like to work out what V is to 2nd order in fluctuations about the equilibrium points. Let these be **R**Ri, and write **r**i = **R**i + **x**i. Then we can expand the two-body potential energy term as... (there is implicit summation over i, j, k, m)



Continuing,



So the total potential energy of our guys would be, presuming the Φ´ terms add up to zero, as it should in equilibrium:



Let’s define,



Then can write:



We can put this in HO form,



We can verify that this formula for K satisfies all the properties enumerated in previous file. For instance, consider the translational symmetry property. We’re supposed to have:



So that checks out. And consider the rotational symmetry property.



And this checks out, since any vector crossed into itself is zero. And we don’t use Ri as the origin, but shift it by some **a**, we still get zero, since:



**Direct Evaluation of K for Potential Energy with pair-wise form**

Let’s do same thing a different way. Recall we said,



where V is the total potential energy. Let Φ be the some pair-wise potential energy between neighbors. And let the interaction extend over all neighbors. Then we would have,



and,



And let’s work this out,



Guess I have to do that sum now. Luckily for us, we can extend the sum to include i = j terms because those are zero. So,



which gives us,



This matches the form of the result we had above.

